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AUTHOR King, Irv
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ABSTRACT

This series of four studies investigated the abilities of children in grades five through eight to perform on a reasoning test written in both Ikonic and Symbolic form. An abstract reasoning test which had been shown to be highly correlated with success in mathematics was rewritten into Ikonic form. Children whose ages bridge the gap between the Ikonic and Symbolic stages of cognitive development were randomly assigned to two groups. The Ikonic test was administered to one group, the Symbolic test to the other. Results showed that the ability to perform on the test increases steadily with age; that the Ikonic form of the test is significantly easier than the Symbolic test, particularly for students of lesser ability; and that there is no evidence to support the hypothesis that the Symbolic test is more highly correlated to success in mathematics (as measured by the SCAT quantitative test) than the Ikonic test. (Author/DT)

IKONIC AND SYMBOLIC REPRESENTATION

A Study of Mathematical Reasoning

by

Irv King

IKONIC AND SYMBOLIC REPRESENTATION

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Irv King

University of Hawaii

Introduction

This study is actually a series of four related studies which investigated certain aspects of Jerome Bruner's theory of learning (Bruner, 1966). A central part of his theory revolves around the postulated existence of levels or stages of cognitive development. According to his theory, until the age of six or seven a child reasons in terms of actions with physical objects (Enactive stage). From the ages of seven to eleven or twelve he is able to reason in terms of pictures and drawings (Ikonic stage). At about the age of twelve or thirteen the child is able to reason abstractly (Symbolic stage). Since knowledge can also be represented in Enactive, Ikonic, and Symbolic modes, Bruner argues that the concepts of any academic discipline can be successfully taught to children at any stage of mental development if we are but clever enough to simplify and reformulate these concepts into modes which match the cognitive structures of the children's minds.

In recent years this approach has been increasingly applied to the mathematics curriculum, as is evidenced by the growing use of concrete material at the elementary school level. The purpose of this study was to use Ikonic and Symbolic forms of knowledge to examine Bruner's theory.

The Test Instrument

Richard Skemp has devised a test which has been shown to be highly correlated to success in learning mathematics (Skemp, 1960). The test defines ten

abstract operations and asks the student to perform various manipulations on these operations. For example, Operation A is "turn the other way up".  becomes  under A. Operation I is "double the smaller part".  becomes  under I. The 45-item test asks the student to perform the following three types of manipulations on these operations:

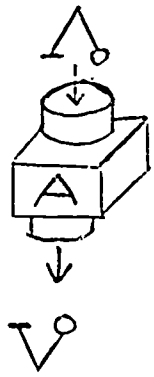
1) Reverse an operation. To reverse I on  would yield  ;

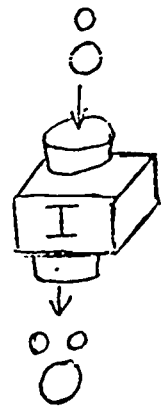
2) Combine two operations. To combine A and I on  would yield  ;

3) Reverse and combine. To reverse and combine A and I on  would yield  .

Skemp believes that this kind of reasoning ability (which he calls "reflective intelligence") is the type of reasoning required for success in mathematics.

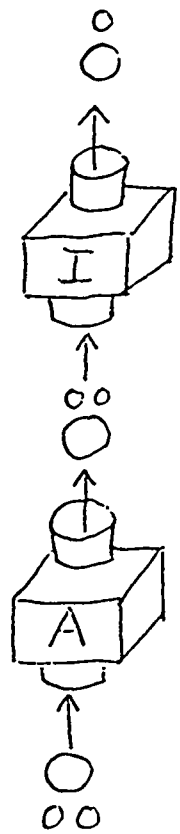
Skemp's test is abstract in nature. However, it can be written in Ikonic form by the use of "function machines", a device used in several new mathematics programs. Each of the ten original operations can be replaced by a function machine. The example cited above can then be interpreted as follows:





You'll recall that Operation A was "turn the other way up" and Operation I was "double the smaller part".

Skemp's test requires the student to reverse operations, combine operations, and to reverse and combine operations. These can be interpreted in terms of machines. For example, to reverse and combine A and I on  could be depicted as follows:



With a symbolic and an ikonic form of Skemp's test available, it is possible to use the tests to investigate two aspects of Bruner's theory of learning. First, we would expect the Ikonic Test to be easier than the Symbolic Test. This is, after all, what Bruner's theory would predict. If it is easier, the result would lend support to the practice of using ikonic devices, such as function machines, to explain mathematical concepts.

Secondly, we would expect pre-adolescent youngsters to have considerably more difficulty with the Symbolic Test than adolescent youngsters. If so, this would lend to support Bruner's theory of the development of cognitive growth.

First Study

The initial study was conducted in a parochial school on Oahu. Students from the fifth through eighth grades were chosen as subjects because these grades bridge the gap between the Ikonic and Symbolic stages of cognitive development. That is, fifth graders are in the Ikonic stage and eighth graders are in the Symbolic stage. Students at each grade level were randomly assigned to take either the Ikonic Test or the Symbolic Test. The 2 x 4 design was completely crossed and balanced with cell size n = 20. Sample size was determined by the fact that there were from 43 to 46 students at each grade level. The following null hypotheses were tested:

H_1: For children in grades 5 through 8, there are no significant differences among the mean scores on Skemp's test obtained by students at different grade levels.

H_2: For children in grades 5 through 8, there are no significant differences between the mean scores obtained by students taking the Ikonic Test and students taking the Symbolic Test.

TABLE 1
Mean Score: Study I

	5th	6th	7th	8th	row means
Ikonic	14.6000	22.2000	20.8500	24.5000	20.5375
Symbolic	11.9000	14.7500	16.5500	18.2500	15.3625
column means	13.2500	18.4750	18.7000	21.3750	

The mean scores are given in Table 1 and the results of the Analysis of Variance is given in Table 2.

TABLE 2
Analysis of Variance: Study I

Source	df	MS	F-Ratio	p
Total	159	84.299		
Between	7	369.972		
A	1	1071.221	15.0572	0.0004
B	3	462.119	6.4956	0.0006
AB	3	44.076	0.6195	0.6074
Within	152	71.143		

Both H_1 and H_2 were rejected at the 0.01 level. The row means show clearly that the Ikonic Test is an easier test than the Symbolic Test. The column means show that the ability to perform on the test increases steadily with age.

Second Study

In addition to searching for differences due to grade level or type of test, in the second study the experimenter calculated the correlation coefficients between each of Skemp's tests and SCAT quantitative ability scores. As in the previous study fifth through eighth grade students served as subjects in a completely crossed and balanced 2 x 4 design. Fourteen subjects were randomly assigned to groups (28 being the smallest

class size). The mean scores are given in Table 3 and the Analysis of Variance is given in Table 4.

TABLE 3
Mean Scores: Study II

	5th	6th	7th	8th	row means
Ikonic	21.1429	23.5714	29.4286	31.1429	26.3214
Symbolic	16.0000	18.3571	27.8571	31.3571	23.3928
column means	18.5714	20.9643	28.6428	31.2500	

TABLE 4
Analysis of Variance: Study II

Source	df	MS	F-Ratio	p
Total	111	106.952		
Between	7	495.609		
A	1	240.133	2.9722	0.0838
B	3	1025.409	12.6919	0.0000
AB	3	50.969	0.6309	0.6006
Within	104	80.793		

Although the Ikonic mean is higher than the Symbolic mean score, H_1 is not rejected by the data. It should be noted that this group of students is far above average the quantitative section of the SCAT test. For the Ikonic group, the mean SCAT score is 73.6250; the mean SCAT score for the Symbolic group was 76.5. This may account for the fact that from the seventh grade on there is little difference between Ikonic means and Symbolic means. That is, perhaps the students at this level can reason well enough to handle either form of the test equally well.

There are clearly differences due to grade level and H_2 is rejected at the 0.01 level. As before, there is evidence of cognitive growth with age.

The correlation coefficient between Ikonic and SCAT scores was computed to be 0.55; the correlation was 0.63 between Symbolic and SCAT scores. Hence the results lend some support to the hypothesis that the Symbolic Test is more highly correlated to mathematical ability than the Ikonic Test.

Third Study

Since the subjects in the first two studies were largely from middle and upper-middle class families, the third study was conducted in an intermediate school whose students were from lower and lower-middle class families. One class at each grade level (7 - 9) were randomly assigned to groups, resulting in a 2 x 3 completely crossed and balanced designed with n = 12. As in the previous study SCAT quantitative scores were correlated with the test results.

The mean scores are listed in Table 5 and the Analysis of Variance is given in Table 6.

TABLE 5
Mean Scores: Study III

	7th	8th	9th	row means
Ikonic	20.0833	28.3333	29.0833	25.8333
Symbolic	11.2500	17.3333	24.0833	17.55555
column means	15.6667	22.8333	26.5833	

TABLE 6
Analysis of Variance: Study III

Source	df	MS	F-Ratio	p
Total	71	129.652		
Between	5	564.188		
A	1	1233.387	12.7505	0.0010
B	2	738.391	7.6333	0.0014
AB	2	55.386	0.5726	0.5721
Within	66	96.732		

Both null hypotheses are rejected at the 0.01 level. The mean SCAT score was 45.9 for the Ikonic group and 45.8 for the Symbolic group. This is thirty points lower than the means for the students in the previous study.

It should be noted that for this group of students the Symbolic Test is considerably more difficult than the Ikonic Test. The implication is that representing ideas in simple form is particularly helpful to students of lesser ability. Both correlation coefficients were very low, 0.28 for Ikonic and SCAT scores and 0.17 for Symbolic and SCAT scores.

Fourth Study

The first three studies revealed that the ability to perform well on the tests increases with age and that students perform significantly better on the Ikonic Test. In the final study the power of the F-test, or the probability of rejecting the null hypothesis when it is false, was controlled by adjusting the sample size.

The power p of an F-test depends on the level of significance at which the test is made, the number of subjects n, and the degree of falsity of the hypothesis being tested. The latter parameter is defined to be the square root of the ratio of the variance of the treatment population means to the variance for error within treatment populations, or

$$\phi' = \sqrt{\frac{\sum_{j}^{K} (\mu_j - \mu)^2 / K}{\sigma^2}}$$

The procedures outlined by Winer (Winer,1962) were applied to a 2 x 4 design. The study was conducted in a public elementary school and a public intermediate school in a middle to lower-middle class neighborhood with fifth through eighth grade students again serving as subjects. For row effects (type of test) $u_j - u = 3$ was considered by the experimenter to be a practically important difference; from previous studies it was estimated that $\sigma^2 = 100$. ϕ' was calculated as follows:

With power set at .90, the level of significance set at = .01, and with two treatment groups, the power function curves indicate that a sample size of 80 is required. For a 2 x 4 design this means a minimum cell size of n = 20. Hence, two classes of students at each grade level were randomly assigned to either an Ikonic or a Symbolic group. This permitted the cell size to increase to 30. As before SCAT quantitative scores were obtained and correlated with the results.

The results are summarized in Tables 7 and 8.

TABLE 7
Mean Scores: Study IV

	5th	6th	7th	8th	row means
Ikonic	18.1333	26.4333	26.8667	31.1000	25.6333
Symbolic	13.8000	18.1000	22.8333	29.3667	21.0250
column means	15.9667	22.2666	24.8500	30.2333	

Both H_1 and H_2 are rejected at the .01 level. The mean SCAT score for the Ikonic group was 65.45 and 69.3417 for the Symbolic group. The correlation coefficients were nearly identical, 0.59 for the Ikonic group and 0.58 for the Symbolic group. (Table 8 found on following page.)

TABLE 9
Analysis of Variance: Study IV

Source	df	MS	F-Ratio	P
Total	239	122.414		
Between	7	1133.036		
A	1	1274.180	13.8617	.0005
B	3	2106.290	22.9142	0.0000
AB	3	112.733	1.2264	0.3002
Within	232	91.921		

Summary

This sequence of four studies investigated the abilities of school children to perform on a reasoning test written in both Ikonic and Symbolic form. An abstract reasoning test which has been shown to be highly correlated with success in mathematics was rewritten into Ikonic form. Children whose ages bridge the gap between the Ikonic and Symbolic stages of cognitive development were randomly assigned to two groups. The Ikonic test was administered to one group, the Symbolic test to the other. In order to collect data from both lower and middle class children, the study was conducted in four different schools.

The analysis of variance performed on the data reveals the following:

1) The ability to perform on the test increases steadily with age. This is as expected and merely reflects the growth of cognitive abilities during the ages from ten through fourteen. The increase in test scores is gradual and does not lend support to a "stage theory" of cognitive development.

2) The Ikonic form of the test is significantly easier than the Symbolic test. This was particularly true for students of lesser ability. This result lends support to the practice of using function machines to help simplify mathematical ideas. Although it would be risky to generalize this particular ikonic device to other attempts to simplify ideas, the results are encouraging. Further studies investigating other ikonic devices need to be undertaken.

3) There is no evidence to support the hypothesis that the Symbolic test is more highly correlated to success in mathematics (as measured by the SCAT quantitative test) than the Ikonic test.

Whereas the results of this study are by no means definitive, they do suggest that curriculum writers and teachers can greatly simplify abstract ideas by using ikonic modes of representation.

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